Modified DEA models without explicite inputs

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Abstract

In performance evaluations, data without explicit inputs (such as index data, pure output data) are widely used. To directly use such data, this paper presents a study on building DEA models without explicit inputs, DEA-WEI models, in which convexity assumption is considered to be satisfied while the ratio data are constructed. Finally, an example of the modified DEA-WEI model is presented and compared to that of conventional one.

Keywords: Data envelopment analysis, Efficiency, convexity, without inputs.

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1-Introduction

Data envelopment analysis (DEA) is a non-parametric method to identify the best-practice frontier rather than the central-tendency, and then only the DMUs on the frontier are classified efficient. In this method, DMUs can freely select their weights to maximize their performance scores. Since the first DEA paper was published in EJOR in 1978, CCR model by Charnes et al. [1], it has become an attractive tool of performance evaluation in both non-profit and for-profit sectors. The standard DEA models have been formulated via input and output data of DMUs. However, as mentioned above, data sets are sometimes given without inputs, or the original input–output data cannot be easily recovered. For example, in an evaluation of research institutes in Chinese Academy of Sciences (CAS), the index data used are publications per staff, research funding per staff, citations per publication and others. It is clearly difficult to recover the original inputs and outputs directly from these indexes as the publications are used both as numerator and denominator here, although in this particular case the original inputs and outputs are in fact available from the CAS database. In some cases, CAS just used the outputs to evaluate the research institutes without explicating considering the inputs at all. Furthermore, in practical applications, often only a part of the indexes is available or meaningful. In performance evaluations, index indicators are widely used in assessment of business, human development, health service, competitiveness or wealth of countries, World competitiveness and others. Let $x_i$ and $y_r$ be the input and output of a decision making unit (DMU), then the index data have the form $e_{ir} = \frac{y_r}{x_i}$. Furthermore, in the evaluation of efficiency or effectiveness such as countries’ power and students’ performance, only outputs are explicitly used. Thus it is sometimes difficult if not impossible to recover the explicit input–output relationship among the data as required in the evaluation applications of the standard DEA models. In practical applications, some aggregation techniques are often employed in order to produce a single score of performance from index data. The most widely used technique is to calculate the weighted sum of indexes. However, how to properly select the weights is a main source of difficulty in the application of this technique. Popular methods to determine the weights include peer review through Delphi or analytic hierarchy process (AHP), statistics methods such as regression analysis and principal component analysis (PCA), and entropy method. The same weights are used for all the DMUs in the above methods. However this is often the source of controversies for the final evaluation results. The aim of this paper is to present more systematic theoretical background for these models in the previous studies.

The paper unfolds as follows. In the next section some preliminaries will be briefly discussed, next the modified approach will be presented and also with a numerical example these two models will be compared and finally section 5 concludes the paper.

2- Preliminaries

Let \{\{Y_j \mid j = 1, \ldots, n\}\} be a group of data in $\mathbb{R}^n$. Then the smallest closed convex and free-disposal attainable set that contains the observations can be further expressed as follows: $AS = \{Y \mid Y \leq Y\lambda, \quad 1\lambda = 1, \quad \lambda \geq 0\}$. Then a DEA model for the observation under evaluation ($Y_0$) is to identify the virtual elements to have the largest residual performance (with a given measure) over $Y_0$. Using the radial measure and the classic arguments of economics, we obtain the DEA-WEI model: $\max{\phi \mid \phi Y_0 \in PPS}$, that is:
\[
\begin{align*}
\text{max} \quad & \phi \\
\text{s.t.} \quad & \sum_{j=1}^{n} \lambda_j y_{rj} \geq \phi y_{r0}, \quad r = 1, \ldots, s, \\
& \sum_{j=1}^{n} \lambda_j = 1, \\
& \lambda_j \geq 0, \quad j = 1, \ldots, n.
\end{align*}
\]

Notice here we do not explicitly consider the input variables in the attainable set. Thus we will only consider bounded attainable sets since otherwise will render infeasibility of the programme—unbounded solutions. In the following proposition, we list some possible ways of building an attainable set. Furthermore, in many applications, it may not be rational to assume radial contraction or expansion. Then one can adopt the Russell measurement, and have the following model:

\[
\begin{align*}
\text{max} \quad & \frac{1}{s} \sum_{r=1}^{s} \phi_r \\
\text{s.t.} \quad & \sum_{j=1}^{n} \lambda_j y_{rj} \geq \phi y_{r0}, \quad r = 1, \ldots, s, \\
& \sum_{j=1}^{n} \lambda_j = 1, \\
& \phi_r \geq 1, \quad r = 1, \ldots, s, \\
& \lambda_j \geq 0, \quad j = 1, \ldots, n.
\end{align*}
\]

If the constraint $\phi_r \geq 1$ is replaced by $\phi_r \geq 0$ in this model, then the average preference is actually used for the outputs. Similarly, one can use DEA-WEI models of SBM type to deal with such applications. Let us note that SBM DEA models can be transformed into Russell DEA models using some variable transformation.

### 3- Modified approach

One of the main assumptions in the definition of efficiency measure underlying DEA is the convexity axiom. Hence many researchers have paid specific attention to the convexity assumption. For example the convex projection approach of Petersen [2], Bogetoft [3], and Bogetoft et al. [4] all dispensed with the assumption of global convexity while at the same time presuming convexity of input and output sets. The ‘convex pairs’ approach of Agrell and Bogetoft [5] dispenses with the input sets and output sets that may themselves be non-convex unions of convex subsets. This approach allows modeling of overall non-convex technologies while retaining convexity in the input and output dimensions in a local sense while Podinovski [6] extended their model in which each input and output is treated individually with respect to the convexity assumption. Let’s take into account the correct convexity for the ratio variables which it should be defined as ratio of convex combination of numerator to the convex combination of denominator rather than a simple convex combination of ratio variable. Consider the data which constitute the desired ratio as n and d. As regards of this definition this means that the convex combination of DMUs should have the kth-output as follows:
Note that in the standard DEA the convex combination is defined as

$$\sum_{j=1}^{n} \lambda_j n_{kj}$$

Therefore the convexity assumption (when assessing unit jo) should be taken into the model as follows:

$$\sum_{j=1}^{n} \lambda_j y_{kj} = \sum_{j=1}^{n} \frac{n_{kj}}{d_{kj}} \lambda_j$$

P Hence model (1) for DMUj0 should be written in the following form:

$$\max \phi$$

s.t. $$\sum_{j=1}^{n} \lambda_j n_{kj} - y_{k0} \phi \sum_{j=1}^{n} \lambda_j d_{kj} \geq 0, \quad r = 1, \ldots, s,$$

$$\sum_{j=1}^{n} \lambda_j = 1,$$

$$\lambda_j \geq 0, \quad j = 1, \ldots, n.$$  

Thus the correct convex combination of the ratio variables, when assessing DMU_{o}, we could conclude a model for input orientation, which is called without output, similarly.

4-Main Approach

In this paper, considering the properties of Rough set theory in the criteria classification and This example compares the results of standard DEA model (2.1) with model (2.3). Consider the following data set with the
same variables as explained in Example (see Table 1). Earlier we have discussed that model (2.1) should not be used in the presence of ratio variables, however if one used model (2.3) the results are clearly incorrect. Table 2 shows the comparison between the two models.

Table.1 Data

<table>
<thead>
<tr>
<th>DMU #</th>
<th>O1</th>
<th>O2</th>
<th>O3</th>
<th>O4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.4</td>
<td>7</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>4.5</td>
<td>4.5</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>3.33</td>
<td>2.5</td>
<td>1</td>
<td>0.75</td>
</tr>
<tr>
<td>4</td>
<td>0.25</td>
<td>0.25</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
<td>0.6</td>
<td>5</td>
<td>1</td>
</tr>
</tbody>
</table>

The results of using models (2.1) and (2.3), respectively, are gathered in Table 2.

Table.2 Results

<table>
<thead>
<tr>
<th>DMU #</th>
<th>model(1)</th>
<th>model(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>1.3</td>
<td>1.3</td>
</tr>
</tbody>
</table>
As it can be seen in the above table, the results obtained through solving models (2.1) and (2.3) are different. Using the DEA models of "without inputs" for the observations containing ratio data as output may result incorrect efficiency scores. A modified DEA models of "without inputs" is presented taking into account the correct convexity of DMUs when a ratio variable is included in the assessment model.

### 4-Example

Data envelopment analysis is a non-parametric method for efficiency assessment of a set of DMUs and it is usually undertaken with absolute numerical data, which among other things reflect the size of the units. There are many cases reported in the literature that DEA is used in the presence of ratio variables. This paper demonstrated that using the DEA models of "without inputs" for the observations containing ratio data as output may result incorrect efficiency scores. A modified DEA model is presented taking into account the correct convexity of DMUs when a ratio variable is included in the assessment model.
References